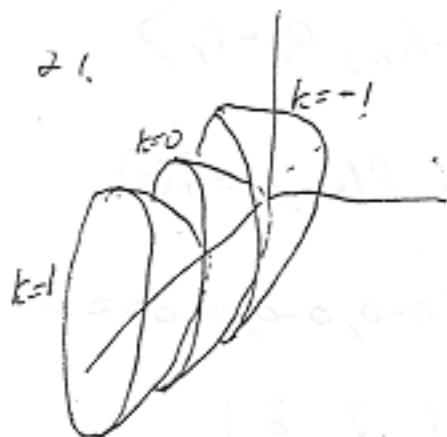


CALCULUS III FINAL EXAM KEY OF MAY 2001

1. C    2. E    3. B    4. C    5. C  
 6. B    7. B    8. D    9. A    10. B  
 11. B    12. A    13. A    14. D    15. A  
 16. A    17. D    18. B    19. C    20. E



22. a.  $L = \int_0^{\pi} \sqrt{[6 \cos(3t)]^2 + 5^2 + [-6 \sin(3t)]^2} dt$   
 $= \int_0^{\pi} \sqrt{61} dt = \boxed{\sqrt{61} \pi} \approx 24.537$

b.  $\vec{r}'(t) = \langle 6 \cos(3t), 5, -6 \sin(3t) \rangle$

$\vec{r}(t) = \langle -2, \frac{5t}{2}, 0 \rangle$  for  $t = \frac{\pi}{2}$ .

$\vec{r}'(\frac{\pi}{2}) = \langle 0, 5, 6 \rangle$

$x = -2, y = \frac{5\pi}{2} + 5t, z = 6t$

23.  $\vec{a}(t) = \langle 0, -32 \rangle$      $\vec{v}(t) = \langle c_1, -32t + c_2 \rangle$      $\langle \frac{41}{2}, \frac{41}{2} \sqrt{3} \rangle = \vec{v}(0) = \langle c_1, c_2 \rangle$  so

$\vec{v}(t) = \langle \frac{41}{2}, -32t + \frac{41}{2} \sqrt{3} \rangle$      $\vec{r}(t) = \langle \frac{41}{2}t + c_3, -16t^2 + \frac{41}{2} \sqrt{3}t + c_4 \rangle$

$\langle 0, 0 \rangle = \vec{r}(0) = \langle c_3, c_4 \rangle$     so  $\vec{r}(t) = \langle \frac{41}{2}t, -16t^2 + \frac{41}{2} \sqrt{3}t \rangle$ .

When  $x=40$ ,  $\frac{41}{2}t = 40$ ,  $t = \frac{80}{41}$ . At that time  $y = -16(\frac{80}{41})^2 + \frac{41}{2} \sqrt{3} \frac{80}{41} \approx 8.37$ .

NO it does not clear the bar.

24.  $g_x = \cos(x)$ ,  $g_y = e^y$      $g_x(0,1) = 1$      $g_y(0,1) = e$      $g(0,1) = e$

$L(x,y) = z_0 + g_x(0,1)(x-0) + g_y(0,1)(y-1) = e + x + e(y-1) = \boxed{x + ey}$ .

$f(0.1, 1.2) \approx L(0.1, 1.2) = 0.1 + 1.2e \approx \boxed{3.36}$

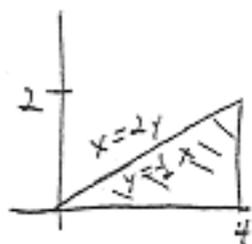
25. a.  $\vec{\nabla} f = \langle 4xy, 2x^2 - 6y \rangle$ ,  $(\vec{\nabla} f)(1,2) = \langle 8, -10 \rangle$ , max rate =  $|\langle 8, -10 \rangle| = \boxed{\sqrt{164}}$

12,806. direction =  $\boxed{\langle 8, -10 \rangle} \parallel \langle \frac{8}{\sqrt{164}}, \frac{-10}{\sqrt{164}} \rangle$

b.  $|\vec{i} + 2\vec{j}| = \sqrt{5}$ ,  $\vec{u} = \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$ .  $(\vec{\nabla} f)(1,2) = \langle 8, -10 \rangle \cdot \langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle = \frac{8}{\sqrt{5}} - \frac{20}{\sqrt{5}}$

$\boxed{\frac{-12}{\sqrt{5}}} \approx -5.3666$

26.



$$\int_{y=0}^2 \int_{x=2y}^4 e^{x^2} dx dy = \int_{x=0}^4 \int_{y=0}^{\frac{1}{2}x} e^{x^2} dy dx$$

$$\text{int}(\text{int}(\exp(x^2), y=0..x/2), x=0..4);$$

27. For  $\vec{F}(x,y,z) = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$ ,

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \langle R_y - Q_z, P_z - R_x, Q_x - P_y \rangle.$$

For  $\vec{F}(x,y,z) = \langle f(x), g(y), h(z) \rangle$  we have  $P(x,y,z) = f(x)$ ,

$$Q(x,y,z) = g(y), \quad R(x,y,z) = h(z), \quad \text{so}$$

$$\text{curl } \vec{F} = \langle h_y - g_z, f_z - h_x, g_x - f_y \rangle = \langle 0 - 0, 0 - 0, 0 - 0 \rangle = \vec{0}.$$

$$28. \vec{r}_u = \langle \cos v, \sin v, 0 \rangle, \quad \vec{r}_v = \langle -u \sin v, u \cos v, 1 \rangle, \quad \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos v & \sin v & 0 \\ -u \sin v & u \cos v & 1 \end{vmatrix} = \langle \sin v, -\cos v, u \rangle$$

$$\iint_S (x^2 + y^2) dS = \int_{u=0}^1 \int_{v=0}^{\pi} ((u \cos v)^2 + (u \sin v)^2) |\vec{r}_u \times \vec{r}_v| dv du = \int_0^1 \int_0^{\pi} u^2 \sqrt{1+u^2} dv du$$

$$= \frac{\pi}{8} (3\sqrt{2} + 4(\sqrt{2}-1)) \approx 1.3200$$

$$29. \text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & x^2 + yz & z \end{vmatrix} = \langle z, 0, 2x \rangle, \quad z = g(x,y) = 3 - 2x$$

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl } \vec{F} \cdot d\vec{S} = \iint_D \langle 3-2x, 0, 2x \rangle \cdot \langle -y, y, 1 \rangle dA = \iint_D \langle 3-2x, 0, 2x \rangle \cdot \langle 2, 0, 1 \rangle$$

$$= \iint_D (6-2x) dA = \int_{\theta=0}^{2\pi} \int_{r=0}^2 (6-2r \cos \theta) r dr d\theta = \boxed{24\pi} \approx 75.398$$

$$30. \text{div } \vec{F} = 3y + 0 - 2z, \quad \iiint_E \vec{F} \cdot d\vec{S} = \iiint_E \text{div } \vec{F} dV = \int_{x=0}^1 \int_{y=0}^{1-x} \int_{z=0}^{1-x-y} (3y-2z) dz dy dx = \boxed{\frac{1}{24}}$$

(or  $-\frac{1}{24}$ ).